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There are no rotating stars of a perfect fluid in Hořava-Lifshitz gravity

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Abstract

Hořava-Lifshitz gravity has covariance only under the foliation-preserving diffeomorphism. This implies that the quantities on the constant-time hypersurfaces should be regular. In this theory, the projectability condition which strongly restricts the lapse function is required. We assume that a star is filled with a perfect fluid, that it has the reflection symmetry about the equatorial plane. As a result, we find that there are no stationary axisymmetric star solutions in Hořava-Lifshitz gravity under the physically reasonable assumptions on the matter sector. This is a serious problem in this theory from the view point of astrophysics. It seems that the invariance under the foliation-preserving diffeomorphism and the projectability condition are questionable since our result is derived from them.

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I. INTRODUCTION

Recently, Hořava proposed a power-counting renormalizable gravitational theory [1, 2]. The theory is called Hořava-Lifshitz gravity because it exhibits the Lifshitz-type anisotropic scaling in the ultraviolet,

$$t \to b^z t, \quad x^i \to b x^i, \tag{1}$$

where t, x^i, b and z are the temporal coordinate, the spatial coordinates, the scaling factor and the dynamical critical exponent, respectively, and i runs over 1, 2 and 3. Since this theory is expected to be renormalizable and unitary, its phenomenological aspects [3, 4] and variants [5, 6] strenuously have been investigated, including black holes [7–10], dark matter [11, 12], dark energy [13], solar system test [14] and so on.

Since higher derivative terms do not contribute at large distances, the action of this theory can recover the apparent form of general relativity if we tune a coupling parameter. In this context, it seems that Hořava-Lifshitz gravity passes astrophysical tests. However, we will show that actually it is not true in this paper.

In this theory, stars have not been studied much [15, 16], while black holes have been investigated eagerly. We also need to investigate the gravitational collapse of stars and the formation of black holes for clear understanding of this theory.

The first study of stars in Hořava-Lifshitz gravity was done by Izumi and Mukohyama [15]. Surprisingly, they found that no spherically symmetric and static solution with a perfect fluid exists in this theory under the assumption that the energy density is a piecewise-continuous and non-negative function of the pressure and that the pressure at the center is positive.

However, it seems that their assumption is too simple to describe realistic stars because of their rotations. In this paper, we investigate a stationary axisymmetric star in Hořava-Lifshitz gravity. We find that the stationary and axisymmetric star with a perfect fluid in reflection symmetry about the equatorial plane does not exist under the physically reasonable conditions on the matter sector. Since we do not use the gravitational action to prove it, our result also works out in other projectable theories, [5, 17]. Our result applies to not only strong gravitational fields like neutron stars but also weak gravitational ones like planets or moons. The non-existence of stationary axisymmetric star solutions gives a serious problem in this theory from the view point of astrophysics. It is worth doubting the invariance under

the foliation-preserving diffeomorphism and the projectability condition since our result is derived from them.

This paper is organized as follows. In Sec. II we shall describe the definitions, the basic equations and the properties of Hořava-Lifshitz gravity. In Sec. III we give the main result that there are no stationary axisymmetric star solutions with a perfect fluid under a set of reasonable assumptions in the matter sector. In Sec. IV we summarize and discuss our result. In appendix A we show the explicit expression for the equation of motion. In appendix B we show the triad components of the extrinsic curvature tensor. In this paper we use the units in which $c = 1$.

II. PROPERTIES OF HOŘAVA-LIFSHITZ GRAVITY

In this section we shall describe the definitions, the basic equations and the properties of Hořava-Lifshitz gravity. Hořava-Lifshitz gravity does not have general covariance since the Lifshitz-type anisotropic scaling treats time and space differently. Instead this theory is invariant under the foliation-preserving diffeomorphism:

$$t \to \tilde{t}(t),\ x^i \to \tilde{x}^i(t, x^j). \tag{2}$$

This means that the foliation of the spacetime manifold by constant-time hypersurfaces has a physical meaning. Thus the quantities on the constant-time hypersurfaces such as the extrinsic curvature tensor and the shift vector must be regular.

The field variables are the lapse function $N(t)$, the shift vector $N^i(t, x)$ and the spatial metric $g_{ij}(t, x)$. Note that the shift vector N^i and the spatial metric g_{ij} depend on both t and x^i but that the lapse function N only does on t. This requirement, called the projectability condition, is imposed on Hořava's original paper [1] from a view point of quantization. It is useful to describe the line element in the Arnowitt-Deser-Misner(ADM) form [18],

$$ds^2 = -N^2dt^2 + g_{ij}(dx^i + N^i dt)(dx^j + N^j dt). \tag{3}$$

The action proposed by Hořava [1] is given by

$$I = I_g + I_m, \tag{4}$$

$$I_g = \int dtd^3x\sqrt{g}N \left\{ \frac{2}{\kappa^2}(K^{ij}K_{ij} - \lambda K^2) - \frac{\kappa^2}{2\omega^4}C_{ij}C^{ij} + \frac{\kappa^2\mu}{2\omega^2}\varepsilon^{ijk}R_{il}D_jR^l_k \right.$$

$$-\frac{\kappa^2\mu^2}{8}R_{ij}R^{ij}+\frac{\kappa^2\mu^2}{8(1-3\lambda)}\left(\frac{1-4\lambda}{4}R^2+\Lambda_W R-3\Lambda_W^2\right)\Bigg\}, \tag{5}$$

where I_m is the matter action, R is the Ricci scalar of g_{ij}, R_{ij} is the Ricci tensor of g_{ij}, D_i is the covariant derivative compatible with g_{ij}, K_{ij} is the extrinsic curvature of a constant-time hypersurface, defined by

$$K_{ij}=\frac{1}{2N}(\partial_t g_{ij}-D_iN_j-D_jN_i), \tag{6}$$

$K=g^{ij}K_{ij}$, C_{ij} is the Cotton tensor, defined by

$$C^{ij}=\varepsilon^{ikl}D_k\left(R^j_l-\frac{1}{4}R\delta^j_l\right), \tag{7}$$

$\varepsilon^{ikl}=\epsilon^{ikl}/\sqrt{g}$ is the antisymmetric tensor which is covariant with respect to g_{ij}, and $\kappa,\omega,\mu,\lambda$ and Λ_W are constant parameters. We can rewrite the gravitational action (5),

$$I_g=\int dtd^3x\sqrt{g}N[\alpha(K^{ij}K_{ij}-\lambda K^2)+\beta C_{ij}C^{ij}+\gamma\varepsilon^{ijk}R_{il}D_jR^l_k+\zeta R_{ij}R^{ij}+\eta R^2+\xi R+\sigma], \tag{8}$$

where parameters $\alpha,\beta,\gamma,\zeta,\eta,\xi$ and σ are given by

$$\begin{aligned}\alpha &= \frac{2}{\kappa^2},\quad \beta=-\frac{\kappa^2}{2\omega^4},\quad \gamma=\frac{\kappa^2\mu}{2\omega^2},\quad \zeta=-\frac{\kappa^2\mu^2}{8},\\ \eta &= \frac{\kappa^2\mu^2}{8(1-3\lambda)}\frac{1-4\lambda}{4},\quad \xi=\frac{\kappa^2\mu^2}{8(1-3\lambda)}\Lambda_W,\quad \sigma=\frac{\kappa^2\mu^2}{8(1-3\lambda)}(-3\Lambda_W^2).\end{aligned} \tag{9}$$

If we take $\lambda=1$ to recover the apparent form of general relativity and the apparent Lorentz invariance, we can compare this action to that of general relativity. Then we obtain

$$\alpha=\frac{1}{16\pi G},\quad \xi=\alpha,\quad \sigma=-2\Lambda\alpha, \tag{10}$$

where Λ is the cosmological constant and G is Newton's constant.

Under the infinitesimal coordinate transformation

$$\delta t=f(t),\qquad \delta x^i=\zeta^i(t,x), \tag{11}$$

g_{ij}, N^i and N transform as

$$\delta g_{ij}=f\partial_t g_{ij}+\mathcal{L}_\zeta g_{ij}, \tag{12}$$

$$\delta N^i=\partial_t(N^if)+\partial_t\zeta^i+\mathcal{L}_\zeta N^i, \tag{13}$$

$$\delta N_i=\partial_t(N_if)+g_{ij}\partial_t\zeta^j+\mathcal{L}_\zeta N_i, \tag{14}$$

$$\delta N=\partial_t(Nf), \tag{15}$$

where $\mathcal{L}_\zeta$ is the Lie derivative along $\zeta^i(t,x)$. $\mathcal{L}_\zeta g_{ij}$ and $\mathcal{L}_\zeta N^i$ are given by

$$\mathcal{L}_\zeta g_{ij} = g_{jk} D_i \zeta^k + g_{ik} D_j \zeta^k, \tag{16}$$

$$\mathcal{L}_\zeta N^i = \zeta^k D_k N^i - N^k D_k \zeta^i. \tag{17}$$

By the variation of the action with respect to N, we get the Hamiltonian constraint

$$H_{g\perp} + H_{m\perp} = 0, \tag{18}$$

where

$$\begin{aligned} H_{g\perp} &\equiv -\frac{\delta I_g}{\delta N} \\ &= \int dx^3 \sqrt{g} \left[(\alpha K^{ij} K_{ij} - \lambda K^2) - \beta C_{ij} C^{ij} - \gamma \varepsilon^{ijk} R_{il} D_j R^l_k - \zeta R_{ij} R^{ij} - \eta R^2 - \xi R - \sigma \right] \end{aligned} \tag{19}$$

and

$$H_{m\perp} \equiv -\frac{\delta I_m}{\delta N} = \int dx^3 \sqrt{g} T_{\mu\nu} n^\mu n^\nu. \tag{20}$$

Here, n^μ is defined as

$$n_\mu dx^\mu = -N dt, \quad n^\mu \partial_\mu = \frac{1}{N}(\partial_t - N^i \partial_i). \tag{21}$$

Notice that due to the projectability condition, $N = N(t)$, the Hamiltonian constraint is global in Hořava-Lifshitz gravity, while it is local in general relativity.

From the variation of the action with respect to N^i, we obtain the momentum constraint

$$\mathcal{H}_{gi} + \mathcal{H}_{mi} = 0, \tag{22}$$

where

$$\mathcal{H}_{gi} \equiv -\frac{1}{\sqrt{g}} \frac{\delta I_g}{\delta N^i} = -2\alpha D^j (K_{ij} - \lambda K g_{ij}), \tag{23}$$

$$\mathcal{H}_{mi} \equiv -\frac{1}{\sqrt{g}} \frac{\delta I_m}{\delta N^i} = T_{i\mu} n^\mu. \tag{24}$$

By the variation of the action with respect to g_{ij}, we get the equation of motion

$$\mathcal{E}_{gij} + \mathcal{E}_{mij} = 0, \tag{25}$$

where

$$\mathcal{E}_{gij} \equiv g_{ik}g_{jl}\frac{2}{N\sqrt{g}}\frac{\delta I_g}{\delta g_{kl}}, \tag{26}$$

$$\mathcal{E}_{mij} \equiv g_{ik}g_{jl}\frac{2}{N\sqrt{g}}\frac{\delta I_m}{\delta g_{kl}} = T_{ij}. \tag{27}$$

The explicit expression for the equation of motion is given in Appendix A.

By the invariance of the gravitational action and the matter action under the infinitesimal transformation (11), we get the energy conservation

$$N\partial_t H_{\alpha\perp} + \int dx^3 \left(N^i\partial_t(\sqrt{g}\mathcal{H}_{\alpha i}) + \frac{N\sqrt{g}}{2}\mathcal{E}_\alpha{}^{ij}\partial_t g_{ij}\right) = 0, \tag{28}$$

and the momentum conservation

$$0 = \frac{1}{N}(\partial_t - N^j D_j)\mathcal{H}_{\alpha i} + K\mathcal{H}_{\alpha i} - \frac{1}{N}\mathcal{H}_{\alpha j}D_i N^j - D^j\mathcal{E}_{\alpha ij}, \tag{29}$$

where α represents g or m .

In the next section, we will only use the momentum conservation of the matter to show that no stationary axisymmetric star solution exists. So our result does not depend on the gravitational action.

III. NO STATIONARY AXISYMMETRIC STAR SOLUTIONS

In this section, we show that there are no stationary axisymmetric star solutions in Hořava-Lifshitz gravity. To prove it, we assume that a star is filled with a perfect fluid, that it has the reflection symmetry about the equatorial plane, that the energy density is a piecewise-continuous and non-negative function of the pressure, that the pressure is a continuous function of r, and that the pressure at the center of the star is positive.

A. Stationary And Axisymmetric Configuration

We consider stationary and axisymmetric configurations with the timelike and spacelike Killing vectors respectively given by

$$t^\mu\partial_\mu = \partial_t, \tag{30}$$

$$\phi^\mu\partial_\mu = \partial_\phi. \tag{31}$$

The timelike Killing vector t^μ implies everywhere

$$N^2 - N_i N^i > 0. \tag{32}$$

The spacelike Killing vector ϕ^μ implies that

$$\phi^\mu \phi_\mu = g_{\phi\phi} \tag{33}$$

is a geometrical invariant.

As a part of the gauge condition, we take

$$g_{r\theta} = g_{r\phi} = 0. \tag{34}$$

Under this gauge condition, the general form for the spatial line element is described by [19]

$$dl^2 = \psi^4 [A^2 dr^2 + \frac{r^2}{B^2} d\theta^2 + r^2 B^2 (\sin\theta d\phi + \xi d\theta)^2], \tag{35}$$

where $\psi, A, B,$ and ξ are functions of r and θ, but neither t nor ϕ for stationarity and axisymmetry.

Now we assume that the spacetime has a rotation axis, where $\sin\theta = 0$. This means

$$\phi^\mu \phi_\mu = g_{\phi\phi} = 0 \tag{36}$$

there [20].

B. Triad Components Of Shift Vector

We define triad basis vectors $\{\boldsymbol{e_{(i)}}\}$. $\boldsymbol{e_{(1)}}$ is along the radial direction, $\boldsymbol{e_{(3)}}$ is along the axial Killing vector and $\boldsymbol{e_{(2)}}$ is fixed by the orthonormality and the right-hand rule. The coordinate components for the orthonormal triad are

$$e^i_{(1)} = \frac{1}{\psi^2} \left[\frac{1}{A}, 0, 0 \right], \tag{37}$$

$$e^i_{(2)} = \frac{1}{\psi^2} \left[0, \frac{B}{r}, -\frac{\xi B}{r \sin\theta} \right], \tag{38}$$

$$e^i_{(3)} = \frac{1}{\psi^2} \left[0, 0, \frac{1}{r B \sin\theta} \right], \tag{39}$$

where we have used the spatial line element (35). The projection of the shift vector on the triad is related to its coordinate components by

$$N_{(1)} = \frac{N_r}{\psi^2 A}, \tag{40}$$

$$N_{(2)} = \frac{N_\theta B}{\psi^2 r} - \frac{N_\phi \xi B}{\psi^2 r \sin\theta}, \tag{41}$$

$$N_{(3)} = \frac{N_\phi}{\psi^2 r B \sin\theta}. \tag{42}$$

C. Regularity Conditions At The Origin

Here we give the regularity conditions of the shift vector N^i near the origin. A tensorial quantity is regular at $r = 0$ if and only if all its components can be expanded in non-negative integer powers of x, y and z in locally Cartesian coordinates, defined by

$$x \equiv r \sin\theta \cos\phi, \tag{43}$$

$$y \equiv r \sin\theta \sin\phi, \tag{44}$$

$$z \equiv r \cos\theta. \tag{45}$$

The Lie derivative of the shift vector N^i along the spacelike Killing vector vanishes, or

$$N^i{}_{,j}\phi^j - \phi^i{}_{,j}N^j = 0. \tag{46}$$

In locally Cartesian coordinates, the spacelike Killing vector is written as

$$\phi^i \partial_i = -y\partial_x + x\partial_y. \tag{47}$$

Then its components of eq.(46) are

$$-N^x{}_{,x}y + N^x{}_{,y}x + N^y = 0, \tag{48}$$

$$-N^y{}_{,x}y + N^y{}_{,y}x - N^x = 0, \tag{49}$$

$$-N^z{}_{,x}y + N^z{}_{,y}x = 0. \tag{50}$$

The general regular solution of these equations is

$$N^x = F_1(z, \rho^2)x - F_2(z, \rho^2)y, \tag{51}$$

$$N^y = F_1(z, \rho^2)y + F_2(z, \rho^2)x, \tag{52}$$

$$N^z = F_3(z, \rho^2), \tag{53}$$

where F_1, F_2 and F_3 are independent and regular functions which depend on z and $\rho^2 \equiv x^2 + y^2$.

Now transforming N^i back to the spherical coordinates r, θ and ϕ, we get the spherical components

$$\frac{N^r}{r} = \sin^2\theta F_1 + \frac{1}{r}\cos\theta F_3, \tag{54}$$

$$\frac{N^\theta}{\sin\theta} = \cos\theta F_1 - \frac{F_3}{r}, \tag{55}$$

$$N^\phi = F_2. \tag{56}$$

On the rotation axis ($\sin\theta = 0$), thus, we obtain

$$N^\theta = 0. \tag{57}$$

Using eqs. (35), (37)-(39) and (54)-(56), the triad components are given by

$$N_{(1)} = \psi^2 A(r\sin^2\theta F_1 + \cos\theta F_3), \tag{58}$$

$$N_{(2)} = \frac{\psi^2}{B}\sin\theta(r\cos\theta F_1 - F_3), \tag{59}$$

$$N_{(3)} = \psi^2 B\sin\theta(r\xi\cos\theta F_1 - \xi F_3 + rF_2). \tag{60}$$

Here we additionally assume the reflection symmetry about the equatorial plane $z = 0$ or $\theta = \pi/2$. Then N^x and N^y must be even functions of z, and N^z must be an odd function of z. This implies that F_1, F_2 must be even functions of z, and F_3 must be an odd function of z. Since N^r is an odd function of z on the rotation axis ($\sin\theta = 0$), we get

$$N^r = 0 \tag{61}$$

at the origin.

D. Matter Sector And Momentum Conservation

For simplicity we assume that the matter consists of a perfect fluid. The stress-energy tensor is given by

$$T_{\mu\nu} = (\rho + P)u_\mu u_\nu + Pg_{\mu\nu}, \tag{62}$$

where P and ρ represent the pressure and the energy density, respectively. We assume the four-velocity given by

$$\begin{aligned} u^\mu \partial_\mu &= \frac{1}{D}(t^\mu + \omega \phi^\mu)\partial_\mu \\ &= \frac{1}{D}\partial_t + \frac{\omega}{D}\partial_\phi, \end{aligned} \tag{63}$$

where

$$D \equiv (N^2 - N_i N^i - 2\omega N_\phi - \omega^2 g_{\phi\phi})^{\frac{1}{2}} \tag{64}$$

is the normalization factor and ω is a function of r and θ. For the four-velocity u^μ to be timelike, we shall have $N^2 - N_i N^i - 2\omega N_\phi - \omega^2 g_{\phi\phi} > 0$.

We set $\alpha = m$, and then the momentum conservation (29) of the matter becomes

$$0 = -\frac{1}{N} N^j D_j (T_{i\mu} n^\mu) + K T_{i\mu} n^\mu - \frac{1}{N} T_{j\mu} n^\mu D_i N^j - D^j T_{ij}. \tag{65}$$

After some calculation, we obtain the r component

$$0 = -P_{,r} + \frac{\rho + P}{D^2} \left\{ \frac{1}{2}(N_i N^i)_{,r} + \omega N_{\phi,r} + \frac{1}{2}\omega^2 g_{\phi\phi,r} + \frac{N_{,r}}{N} N^r N_r + \frac{N_{,\theta}}{N} N^\theta N_r \right\}. \tag{66}$$

Now we use the projectability condition $N = N(t)$ and this equation becomes

$$0 = -P_{,r} + \frac{\rho + P}{D^2} \left\{ \frac{1}{2}(-N^2 + N_i N^i)_{,r} + \omega N_{\phi,r} + \frac{1}{2}\omega^2 g_{\phi\phi,r} \right\}. \tag{67}$$

We do not use the θ and ϕ components to prove that no stationary axisymmetric star exists.

Here we concentrate on the r component of the momentum conservation of the matter on the rotation axis $\sin\theta = 0$. On the rotation axis, $g_{\phi\phi}$ and $g_{\phi\phi,r}$ vanish from eq. (36). From eq. (42), the regularity of the triad component of the shift vector $N_{(3)}$ implies

$$N_\phi = 0 \tag{68}$$

on the rotation axis. Thus $N_{\phi,r} = 0$. Thus the r component of the momentum conservation (67) on the rotation axis becomes

$$0 = -P_{,r} - \frac{1}{2}\frac{(\rho + P)(N^2 - N_i N^i)_{,r}}{N^2 - N_i N^i}. \tag{69}$$

E. Contradiction Of Momentum Conservation

We assume that the star has the reflection symmetry about the equatorial plane $\theta = \frac{\pi}{2}$, that the energy density ρ is a piecewise-continuous and non-negative function of the pressure P, that the pressure P is a continuous function of r [21] and that the pressure at the center of the star $P_c \equiv P(r=0)$ is positive. Thus, $\rho + P$ is a piecewise-continuous function of r. We have assumed that the energy density ρ is non-negative everywhere and that the pressure at the center P_c is positive, hence $\rho + P$ is positive at the center. We define r_s as the minimal value of r for which at least one of $(\rho+P)|_{r=r_s}$, $\lim_{r\to r_s - 0}(\rho + P)$ and $\lim_{r\to r_s+0}(\rho+P)$ is nonpositive.

Dividing the momentum conservation (69) by $\frac{1}{2}(\rho+P)$, we have

$$\left\{\log\left(N^2 - N_i N^i\right)\right\}_{,r} = -2\frac{P_{,r}}{\rho + P}. \tag{70}$$

Under the assumption that the energy density is a function of the pressure, $\rho = \rho(P)$, integrating the momentum conservation (70) over the interval $0 \le r < r_s$, we obtain

$$\log\left(N^2 - N_i N^i\right)\big|_{r=r_s} - \log\left(N^2 - N_i N^i\right)\big|_{r=0} = -2\int_{P_c}^{P_s} \frac{dP}{\rho+P}, \tag{71}$$

where $P_s \equiv P(r = r_s)$.

The definition of r_s implies that at least one of $(\rho+P)|_{r=r_s}$, $\lim_{r\to r_s-0}(\rho+P)$ and $\lim_{r\to r_s+0}(\rho+P)$ is nonpositive. Since we have assumed that $P(r)$ is a continuous function and that ρ is non-negative everywhere, $P_s = \lim_{r\to r_s-0} P = \lim_{r\to r_s+0} P$ is non-positive. Thus, we get

$$P_s \le 0 < P_c. \tag{72}$$

This implies that the right-hand side of eq. (71) is positive. However, the left-hand side of eq. (71) is nonpositive since we have the projectability condition $N = N(t)$ and we obtain from eqs. (57), (61) and (68)

$$N_i N^i\big|_{r=0} = 0 \tag{73}$$

at the center of the star. This contradicts that the right-hand side of eq. (71) is positive.

IV. DISCUSSION AND CONCLUSION

Hořava-Lifshitz gravity is only covariant under the foliation-preserving diffeomorphism. This means that the foliation of the spacetime manifold by the constant-time hypersurfaces has a physical meaning. As a result, the regularity condition at the center of a star is more restrictive than the one in a theory which has general covariance.

Under the assumption that a star is filled with a perfect fluid, that it has the reflection symmetry about the equatorial plane, and that the matter sector obeys the physically reasonable conditions, we have shown that the momentum conservation is incompatible with the projectability condition and the regularity condition at the center for stationary and axisymmetric configurations. Since we have not used the gravitational action to prove it, our result is also true in other projectable theories [5, 17]. Note that our result is true under not only strong-gravity circumstances like neutron stars but also weak-gravity ones like planets or moons. However, it is not certain that star solutions can exist in non-projectable theories. Since we have used both the covariance under the foliation-preserving diffeomorphism and the projectability condition to prove the non-existence of stationary axisymmetric star, our proof will does not apply if we assume just one of the two.

Greenwald, Papazoglou and Wang found static spherically symmetric solutions with a perfect fluid plus a heat flow along the radial direction under the assumption that the spatial curvature is constant in a projectable theory without the detailed balance condition [16], although it is doubtful that the constant-spatial-curvature solutions represent realistic stars. This, however, implies that rotating star solutions with a perfect fluid plus a heat flow can also exist. We might get solutions by introducing a heat flow or an exotic matter with a negative pressure, but it seems that the physical justification to introduce it is difficult.

Izumi and Mukohyama found that no spherically symmetric and static solution with a perfect fluid exists in this theory under the assumption that the energy density is a piecewise-continuous and non-negative function of the pressure and that the pressure at the center is positive [15]. They concluded that a spherically symmetric star should include a time-dependent region near the center. Although we cannot deny that stars should be described by dynamical configurations, we emphasize that non-existence of stationary axisymmetric star solutions is an unattractive feature of this theory.

The non-existence of stationary axisymmetric star solutions gives a serious problem in

this theory from the view point of astrophysics. It seems that the invariance under the foliation-preserving diffeomorphism and the projectability condition are questionable since our result is derived from them.

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APPENDIX A. EXPLICIT EXPRESSION FOR EQUATION OF MOTION

After a long straightforward calculation, we obtain the explicit expression for the equation of motion

$$
\begin{aligned}
& \alpha\left[\frac{N}{2}K^{lm}K_{lm}g^{ij} - 2NK^{im}K^j_m - \frac{1}{\sqrt{g}}(\sqrt{g}K^{ij}) - D_p(K^{ip}N^j) - D_p(K^{pj}N^i) + D_p(K^{ij}N^p)\right] \\
-\ & \alpha\lambda\left[\frac{N}{2}K^2g^{ij} - 2NKK^{ij} - \frac{1}{\sqrt{g}}(\sqrt{g}Kg^{ij}) - D_p(Kg^{ip}N^j) - D_p(Kg^{jp}N^i) + D_p(KN^pg^{ij})\right] \\
+\ & \beta\Big[-\frac{1}{2}NC^{kl}C_{kl}g^{ij} + 2NC^{jl}C^i_l + 2\varepsilon^{pkl}R^j_lD_k(NC^i_p) - \varepsilon^{pki}D_mD^jD_k(NC^m_p) \\
-\ & \varepsilon^{pkl}D_lD^jD_k(NC^i_p) + \varepsilon^{pkj}D^lD_lD_k(NC^i_p) + \varepsilon^{pkl}g^{ij}D_mD_lD_k(NC^m_p) \\
-\ & \varepsilon^{kil}D_p(NC^j_kR^p_l) - \varepsilon^{pkl}D_k(NC^j_pR^i_l) + \varepsilon^{pil}D_k(NC^k_pR^j_l)\ \Big] \\
+\ & \gamma\Big[\varepsilon^{pqk}D_pD^i(ND_qR^j_k + \frac{1}{2}R^j_kD_qN) + \varepsilon^{jqk}D_lD^i(ND_qR^l_k + \frac{1}{2}R^l_kD_qN) \\
-\ & \varepsilon^{iqk}D^lD_l(ND_qR^j_k + \frac{1}{2}R^j_kD_qN) - \varepsilon^{pqk}g^{ij}D_pD_l(ND_qR^l_k + \frac{1}{2}R^l_kD_qN) \\
+\ & \varepsilon^{pqk}R^j_kD_q(ND_qR^i_p) + \varepsilon^{ikp}D_l(NR^l_pR^j_k)\Big] \\
+\ & \zeta\left[\frac{1}{2}NR_{kl}R^{kl}g^{ij} - 2NR^{il}R^j_l + 2D_kD^j(NR^{ki}) - D^lD_l(NR^{ij}) - g^{ij}D_kD_l(NR^{kl})\right] \\
+\ & \eta\left[\frac{1}{2}NR^2g^{ij} - 2NRR^{ij} + 2D^iD^j(NR) - 2g^{ij}D^lD_l(NR)\right]
\end{aligned}
$$

$$+\ \xi\left[\frac{1}{2}NRg^{ij}-NR^{ij}+D^jD^iN-g^{ij}D^lD_lN\right]+\sigma N\frac{1}{2}g^{ij}+(i\leftrightarrow j)+\frac{2}{\sqrt{g}}\frac{\delta I_m}{\delta g_{ij}}=0, \tag{A.1}$$

where $(i\leftrightarrow j)$ means the terms exchanged i and j each other.

APPENDIX B. TRIAD COMPONENTS OF EXTRINSIC CURVATURE TENSOR

In this theory, the triad components of the extrinsic curvature tensor also should be regular. The Lie derivative of g_{ij} along N^i is

$$\begin{aligned}\mathcal{L}_{\boldsymbol{N}}g_{ij} &= D_jN_i+D_iN_j\\ &= g_{ik}N^k_{\ ,j}+g_{jk}N^k_{\ ,i}+g_{ij,k}N^k.\end{aligned} \tag{B.1}$$

The extrinsic curvature tensor (6) and eq. (B.1) yield

$$\frac{dg_{ij}}{dt}-N^k_{\ ,i}g_{jk}-N^k_{\ ,j}g_{ki}=2NK_{ij}, \tag{B.2}$$

where

$$\frac{d}{dt}\equiv\frac{\partial}{\partial t}-N^i\frac{\partial}{\partial x^i}. \tag{B.3}$$

By projecting (B.2) onto the triad (37)-(39) , we obtain the following equations [19]

$$NK_{(1)(1)}=-N^r_{\ ,r}+\frac{1}{A}\frac{dA}{dt}+\frac{2}{\psi}\frac{d\psi}{dt}, \tag{B.4}$$

$$\frac{2NK_{(1)(2)}}{\sin\theta}=\frac{AB}{r}N^r_{\ ,X}-\frac{r}{AB\sin\theta}N^\theta_{\ ,r}, \tag{B.5}$$

$$\frac{2NK_{(1)(3)}}{\sin\theta}=-\frac{rB}{A}[N^\phi_{\ ,r}+\frac{\xi}{\sin\theta}N^\theta_{\ ,r}], \tag{B.6}$$

$$NK_{(2)(2)}=\frac{1}{r}\frac{dr}{dt}+\frac{2}{\psi}\frac{d\psi}{dt}-\frac{1}{B}\frac{dB}{dt}-N^\theta_{\ ,\theta}, \tag{B.7}$$

$$NK_{(3)(3)}=\frac{1}{r}\frac{dr}{dt}+\frac{2}{\psi}\frac{d\psi}{dt}+\frac{1}{B}\frac{dB}{dt}-\frac{\cos\theta}{\sin\theta}N^\theta, \tag{B.8}$$

$$2NK_{(2)(3)}=B^2\frac{d\xi}{dt}+(1-X^2)B^2(N^\phi_{\ ,X}+\frac{\xi}{\sin\theta}N^\theta_{\ ,X}), \tag{B.9}$$

where

$$X\equiv\cos\theta. \tag{B.10}$$

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[21] In fact, the pressure discontinuity can be excluded from the regularity of the extrinsic curvature as in Ref. [15], if we can admit Eq. (69).